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MAL-003-001515

Seat No. _____

B. Sc. (Sem. V) (CBCS) Examination

October / November – 2016

Mathematics : Paper-BSMT-503-(A) (Theory)

[Discrete Mathematics & Complex Analysis-1]

Faculty Code : 003

Subject Code : 001515

Time : $2\frac{1}{2}$ Hours] [Total Marks : **70**

Instructions : (i) All the questions are compulsory.

(ii) Numbers written to the right indicate full marks of the question.

1 Answer all the questions : **20**

(1) Write an example of a relation on $A = \{a, b, c\}$ which is symmetric and transitive but not reflexive.

(2) Write distributive inequalities.

(3) Write the maximal elements of the POSET ({2,3,4,6},D).

(4) Define : Complemented lattice.

(5) 'R is not reflexive means that R is irreflexive'. True or False ?

(6) Define : Atom of Boolean Algebra.

(7) Define : Sub Boolean Algebra.

(8) If a and b are distinct atoms of the Boolean Algebra $(B, *, \oplus, ', 0, 1)$ then $a * b =$ _____.

(9) Write atoms of Boolean algebra $(S_{30}, *, \oplus, ', 0, 1)$.

(10) Define : Meet and Join in a lattice $(L, \leq)$.

(11) Define : Complex function.

(12) Evaluate : $\lim_{Z\to\infty} \frac{2z+3}{z+i}$.

(13) Define : Analytic function.

(14) Write Laplace equation in polar form.

(15) Define : Closed curve.

(16) Write the formula to obtain the length of a smooth arc.

(17) State Cauchy's fundamental theorem.

(18) Define Entire function.

(19) Write formula for $f'(z)$ in Cartesian form.

(20) What is the imaginary part of $(\sin x + i\cos x)^7$.

2 (a) Attempt any three : **6**

(1) If $(L, *, \oplus, 0, 1)$ is a bounded lattice then prove that

(i) $a*1=a$ (ii) $a\oplus 1=1$.

(2) Which of the following subsets of P(S) for S={1,2,3} are covering or partition of S ?

(i) A={{1,2,3}}

(ii) B={{1,2}, {2,3}}

(iii) C={{1},{1,2},{1,3}}

(iv) D={{1},{2},{3}}

(3) Draw the Hasse diagram of (S_{60}, D).

(4) If $(B, *, \oplus, ', 0, 1)$ is a Boolean algebra then prove that

$\forall a, b, c \in B$,

$(a*b)\oplus(b*c)\oplus(c*a)=(a\oplus b)*(b\oplus c)*(c\oplus a)$.

(5) Let $(B, *, \oplus, ', 0, 1)$ be a finite Boolean algebra. If x is a non-zero element of B then prove that there exists an atom $a \in B$ such that $a \le x$.

(6) State and prove absorption property of a Lattice.

(b) Attempt any three : **9**

(1) Show that similarity of matrices on the set on $n\times n$ matrix is an equivalence relation.

(2) Let $(L,*,\oplus)$ be a lattice. For $a,b \in L$ define a relation R in L as $aRb \Leftrightarrow a*b=a$ then prove that R is a partial order relation in L.

(3) State and prove modular inequality.

(4) If $(B, *, \oplus, ', 0, 1)$ be a finite Boolean algebra then prove that $\forall a,b \in B$

$a \leq b \Leftrightarrow a*b'=0 \Leftrightarrow b' \leq a' \Leftrightarrow a' \oplus b = 1$.

(5) State and prove D'Morgan's law for the Boolean algebra.

(6) Define product of sum canonical form and obtain the product of sum canonical form of $\alpha(x_1,x_2,x_3)=x_1x_2$.

(c) Attempt any two : **10**

(1) Define cube array representation of a Boolean function and find the cube array representation of the function $f(x,y,z)=xy+xz'$.

(2) If $(B, *, \oplus, ', 0, 1)$ is a Boolean algebra then prove that any $x_1, x_2 \in B$.

(i) $A(x_1 * x_2) = A(x_1) \cap A(x_2)$

(ii) $A(x_1 \oplus x_2) = A(x_1) \cup A(x_2)$

(3) If $(L,*,\oplus)$ be a lattice then for any $a,b \in L$ prove that $\text{glb}\{a,b\} = a*b$ and $\text{lub}\{a,b\} = a \oplus b$ with respect to the partial ordering R on L.

(4) Define direct product of two lattices and prove that direct product of two lattices is also a lattice.

(5) State and prove Stone's Representation theorem of Boolean Algebra.

3 (a) Attempt any three : **6**

(1) Prove that e^z is analytic.

(2) Define limit and differentiability of a complex function.

(3) Show that $f(z)=z^2$ is an entire function.

(4) Find $\int_C \frac{z^2}{(z-1)} dz;\ C: |z| = 2$.

(5) Find $\int_C z^2 dz$, where C is the arc of $y = x^2$ from point $z = 0$ to $z = 2 + i$.

(6) State :

(i) Green's theorem (ii) Fundamental theorem of Algebra.

(b) Attempt any three : **9**

(1) Evaluate : $\int_C \frac{z}{(9-z)^2 (z+i)} dz;\ C: |z| = 2$.

(2) Show that $f(z) = (3x + y) + i(3y - x)$ is an entire function.

(3) Prove that $\cos z$ is analytic function and find $f'(z)$.

(4) State and prove Cauchy's inequality.

(5) Prove that $u = r^2 \cos 2\theta$ is harmonic function and find its conjugate.

(6) State and prove Liouville's theorem.

(c) Attempt any two : **10**

(1) Obtain C-R conditions for an analytic function *f*(*z*) in polar form.

(2) Show that $u = \frac{1}{2} \log\left(x^2 + y^2\right)$ is harmonic and find its conjugate.

(3) State and prove Cauchy integral formula.

(4) Evaluate : $\int_C \frac{z^3 + 3}{z^2 (z-4)} dz;\ C: |z| = 1$.

(5) State and prove Morera's theorem.